Mathematical Morphology and Applications - End-Sem Exam

B.Math.(Hons.) Third Year

May 5, 2017

Instructions: There are **7 questions** altogether. Marks corresponding to each question is indicated in bold. Answer as many as you can. Maximum score : **60 marks**. Maximum time : **3 hrs**.

(1) (a) Define Serra's method to calculate the morphological median.

(b) Calculate the dual median of the definition above.

(c) Characterize the difference between the median and dual median.

[3+3+3]

(2) Suppose X is a 2-D $n \times n$ binary image on a square grid. One can implement a dilation of X w.r.t $kB = B \oplus B \oplus \cdots \oplus B$ (k times) or a multi-scale dilation of X w.r.t. B where $B = \{(-1,0),(0,-1),(0,0),(0,1),(1,0)\}$ using either Algorithm 1 k successive times or Algorithm 2 (see below).

(a) Complete Algorithm 3 so that it has $\mathcal{O}(n^2)$ time complexity and $\mathcal{O}(1)$ space complexity. Also, justify the correctness of your algorithm.

(b) Which among the following is more efficient: implementing Algorithm 1 k times or Algorithm 2? You will have to justify your answer by comparing the asymptotic time and space complexities (note that these expressions are functions of both n and k).

Algorithm 1 A Dilation Algorithm

```
1:  procedure DILATE(X, B)                                  ▷ The dilation of X by B
2:      l ← X.shape[0] and b ← X.shape[1]    ▷ Identify the length and width of the image X
3:      for i < l do                                        ▷ Iterating over all the rows
4:          for j < b do                                    ▷ Iterating over all the columns
5:              if X[i][j] == 1 then     ▷ Lines 5-13: Marking the non-foreground pixels adjacent to
    foreground pixels
6:                  if i > 0 and X[i − 1][j] = 0 then
7:                      X[i − 1][j] = 2
8:                  if j > 0 and X[i][j − 1] = 0 then
9:                      X[i][j − 1] = 2
10:                 if i < l − 1 and X[i + 1][j] = 0 then
11:                     X[i + 1][j] = 2
12:                 if j < b − 1 and X[i][j + 1] = 0 then
13:                     X[i][j + 1] = 2
14:     for i < l do                                        ▷ Iterating over all the rows
15:         for j < b do                                    ▷ Iterating over all the columns
16:             if X[i][j] == 2 then
17:                 X[i][j] = 1
18:     return X                                            ▷ Outputs the dilated image X
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[8+4]

Algorithm 2 A Multiscale Dilation

procedure MULTISCALEDILATE(X, B, k) ▷ The dilation of X by kB
  $l \leftarrow X.shape[0]$ and $b \leftarrow X.shape[1]$ ▷ Identify the length and width of the image X
  X = TAXICAB(X) ▷ Compute the distances of every pixel from a nearest foreground pixel w.r.t taxicab metric
  for $i < l$ **do** ▷ Iterating over all the rows
    for $j < b$ **do** ▷ Iterating over all the columns
      if $X[i][j] \leq k$ **then**
        $X[i][j] = 1$
      else
        $X[i][j] = 0$
  return X ▷ Outputs the multi-scale dilated image X

Algorithm 3 Distances from Nearest Foreground Pixel

procedure TAXICAB(X) ▷ The distances of every pixel in X from a nearest foreground pixel w.r.t. taxicab metric
  ...
  ...
  ...
  return X ▷ Outputs a 2-D array X containing the nearest distances from foreground

(3) Let B be a convex set in $\mathcal{P}(\mathbb{R}^2)$. Define

$$\mathcal{L} = \{X \oplus B, \text{ for all } X \subseteq \mathbb{R}^2\}$$

Define the order relation as - $X \leq Y$ if and only if $X \subseteq Y$. Is $(\mathcal{L}, \leq)$ a lattice? If yes, characterize the sets $X \wedge Y$ and $X \vee Y$ for any two sets $X, Y \in \mathcal{L}$.

[4+(2+2)]

(4) Recall that the definition of dilation $\delta(.)$ on $\mathcal{P}(\mathbb{R}^2)$ is given by

$$\delta_B(X) = \bigcup_{b \in B} X_b$$

Dilation can also be defined on a lattice $(\mathcal{L}, \leq)$ as the increasing operator which preserves the supremum. i.e.,

$$\delta\left(\bigvee_i x_i\right) = \bigvee_i \delta(x_i)$$

Stating the minimum required assumptions, show the equivalence between these two definitions. What can be said about the erosion operator?

[6+3]

(5) Let X be a disconnected compact set in $\mathbb{R}^2$ w.r.t. euclidean metric. Define the $SKIZ(.)$ operator on X and assume that for every point $x \in SKIZ(X)$, there is an associated distance $d(x) \in \mathbb{R}^+$ which denotes the distance from x to the nearest point in X. Prove or disprove:

$$X^c = \bigcup_{x \in SKIZ(X)} B(x, d(x))$$

where $B(x, d)$ is the closed ball with center x and radius d.

[5]

(6) (a) Define a white ASF (alternating sequential filter) and describe some cases where ASF is preferable over multi scale openings.

(b) Can a white/black ASF and Id (identity) operators be compared? If yes state the relation and prove it. If not explain with the help of counter examples.

[(1+3)+(1+3)]

(7) Indicate *True* or *False* for each of the following statements and provide reasons supporting your answers. You will have to prove the statement if you indicate *True* and provide a counter example if you indicate *False*. Correct answer carries 1 mark each and a valid reasoning carries 4 marks each.

(a) Let $\mathcal{G}_1 = (V, E, W_1)$ be a vertex-weighted graph. Suppose $\mathcal{G}_2 = (V, E, W_2)$ is another vertex-weighted graph such that $W_2(x) = W_1(x) - 1$ where x is a destructible point of $\mathcal{G}_1$ and $W_2(y) = W_1(y)$ for every $y \in V \setminus \{x\}$. Let $M(\mathcal{G})$ denote the minima of $\mathcal{G}$ (which is a sub-graph of $\mathcal{G}$). Assume that $M(\mathcal{G}_1)$ has at least two connected components. The pass value between a given pair of connected components of $M(\mathcal{G}_1)$ in $\mathcal{G}_1$ is same as the pass value between the same pair in $\mathcal{G}_2$.

(b) Suppose $\mathcal{G} = (V, E, W)$ is a vertex-weighted graph then topological watershed of $\mathcal{G}$ is unique.

(c) Suppose $\mathcal{G} = (V, E, W)$ be an edge-weighted graph. Let M denote the minima of the graph $\mathcal{G}$. A watershed cut i.e. a cut induced by a minimum spanning forest relative to M is a max-cut relative to M.

[3 × (1+4) = 15]